Radiation-Resisted Shock Waves in Gas-Particle Flows

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The effect of thermal radiation on the structure of normal shock waves in gas-particle flows is analyzed. The study is restricted to conditions at which the gas flow can be decoupled from the particle flow and where the gas contribution to radiation is negligible. An approximate closed-form solution, derived for a gray, absorbing particle cloud, demonstrates that, within the limits of the analysis and for a given shock Mach number, radiative influences increase with increasing stagnation temperature and particle diameter and decreasing pressure and particle loading. The radiation-affected regions were found to extend large distances upstream of the gas phase discontinuity. Results from numerical calculations are in substantial agreement with those of the approximate theory. Decreasing the shock Mach number, at constant pressure, stagnation temperature, particle loading, and particle diameter, increased the fraction of the particle temperature rise across the shock occurring upstream of the gas phase discontinuity.

Nomenclature

A,B,C	= constants appearing below Eq. (14)
a,b,c	= constants appearing below Eq. (23)
B_0	= Boltzmann number
C_D	= particle drag coefficient
c_P	= constant pressure specific heat of gas
c_S	= specific heat of particle material
d_P	= particle diameter
k	= thermal conductivity of gas
M	= Mach number of gas in shock-fixed coordinates
M_R	= Mach number of particle relative to gas
Nu	= Nusselt number
n_P	= particle number density
p	= pressure
Pr	= Prandtl number
q_r	= net radiative heat flux
Re	= particle Reynolds number
T	= gas temperature
T_P	= particle temperature
u	= gas velocity in shock-fixed coordinates
u_P	= particle velocity in shock-fixed coordinates
X	= downstream distance in shock-fixed coordinates
α	= gas volumetric absorption coefficient
α_P	= particle cloud volumetric absorption coefficient
ϵ	= particle-to-gas mass flow ratio
γ	= ratio of specific heats
$\bar{\eta}$	= nondimensional radiative flux gradient, Eq. (18)
ρ	= mass density of gas
ρ_P	= mass density of particle cloud
σ	= Boltzmann's constant
ξ	= nondimensional distance defined by Eq. (11)

Subscripts

1	= immediately upstream of gas phase discontinuity
2	= immediately downstream of gas phase discontinuity
+	= region downstream of gas phase discontinuity
−	= region upstream of gas phase discontinuity
$+\infty$	= downstream infinity
$-\infty$	= upstream infinity

Superscript

$\overline{(\ )}$	= nondimensional quantity

Introduction

THE structure of shock waves in gas flows where thermal radiation provides a significant mode of energy transfer has been extensively studied.[1,2] A number of investigations have also been undertaken to determine the effect on shock structure produced by the addition of micron-sized particles to the flow.[3,4] Because particle clouds can be good absorbers and emitters of thermal radiation, it would be of interest to study the effect of radiation on shock structure in gas-particle flows. This paper presents some results of an analytical study of normal shock structure in radiating flows of gray, absorbing gas-particle mixtures restricted to cases where the fraction of radiative flux absorbed by the gas is much smaller than that absorbed by the particle cloud. It will be demonstrated that, even for small particle-to-gas mass flow ratios, a rather broad range of flow conditions exist where such a restriction is satisfied and within which significant radiative effects on the flow can be found.

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Assumptions

Generally accepted assumptions made with regard to gas-particle flows will be adopted in the present analysis.[5] The following assumptions were either necessitated by the problem restriction, appeared valid whenever significant radiative influence is present, or resulted in simplification in solution without qualitatively affecting the final results. 1) The gas is perfect having a constant specific heat. 2) The particles are uniformly sized spheres, have a constant specific heat, and possess a uniform temperature. 3) Particle drag and heat transfer do not affect the gas flow. 4) Low Reynolds number

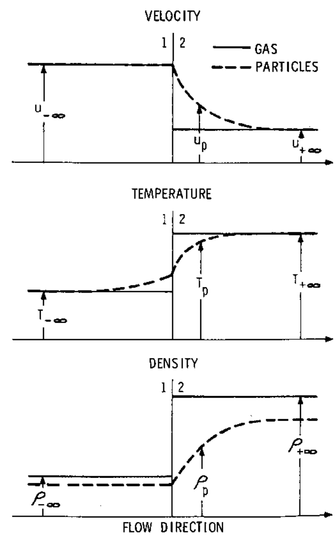

Fig. 1 Assumed shock structure.

drag and heat-transfer laws, corrected for rarefaction effects, may be used to represent the effect of the gas flow on the particle flow. 5) The particle velocity and temperature do not change across the gas phase discontinuity. 6) The gas-particle mixture is a gray, absorbing medium. 7) The volumetric absorption coefficient of the gas is much smaller than that of the particle cloud.

The structure of a normal shock wave in a flow where the foregoing restrictions are satisfied, and at conditions where significant radiative effects are present, is sketched in Fig. 1. The shock is seen to consist of 1) regions, upstream and downstream of the gas phase discontinuity, where the gas conditions are uniform and related through the conventional normal shock equations; 2) a relaxation zone, downstream of the gas phase discontinuity, where drag and heat transfer drive the particle velocity and temperature towards the respective gas values; and 3) a zone, upstream of the gas phase discontinuity, where the particle temperature increases due to the absorption of radiation emitted by hotter particles further downstream. Since the particle flow is assumed to have no effect on the gas flow, the gas velocity, and consequently the particle velocity, are constant upstream of the gas phase discontinuity.

Significance of Gas Radiation

Assumption 7 assures the absorption of radiation by the gas will be negligibly smaller than that absorbed by the particles. This assumption is certainly not valid for all gas compositions at arbitrarily selected temperatures and pressures. To demonstrate that flow conditions and gas-particle mixtures exist, however, where such an assumption is satisfied and at which significant radiative effects can be found, we will compare the particle cloud absorption coefficient modeled by[6]

$$\alpha_P = \pi d_P{}^2 n_P/4 = 3\rho_P/(2\rho_s d_P) \tag{1}$$

with the absorption coefficient for air conservatively approximated by[7]

$$\alpha \approx 0.2\ (\rho/\rho_0)(T/10^4)^5\ \mathrm{cm}^{-1};\ T < 10^{4}\,^\circ\mathrm{K} \tag{2}$$

where $\rho_0 = 1.29 \times 10^{-3}$ gm/cc and with T in °K.

Dividing Eq. (2) by Eq. (1) to obtain

$$\alpha/\alpha_P = \approx 0.133\ (\rho_s d_P/\rho_P)(\rho/\rho_0)(T/10^4)^5 \tag{3}$$

it may be noted that the highest fraction of radiation absorbed by the gas will occur immediately downstream of the gas phase discontinuity, where, in accordance with our assumptions, $\rho = \rho_{+\infty}$, $\rho_P = \rho_{P-\infty}$, and $T = T_{+\infty}$. Thus, with ρ_s in gm/cc, d_P in microns, and $\rho_{P-\infty} = \epsilon\rho_{-\infty}$, we have

$$(\alpha/\alpha_P)_{\max} \approx 0.01\ (\rho_s d_P/\epsilon)(\rho_{+\infty}/\rho_{-\infty})[(T_{+\infty}/T_0)(T_0/10^4)]^5 \tag{4}$$

Equation 4 demonstrates that, for shock Mach numbers to infinity, particle-to-gas mass flow ratios in excess of 0.1, particle mass densities to 4 gm/cc, particle diameters to 5μ, and stagnation temperatures to 4000°K, the maximum absorption of radiation by air will be at least an order of magnitude smaller than that absorbed by the particle cloud. The analysis to follow will show that, under such conditions, radiation can have significant effects on normal shock structure in gas-particle flows.

Analysis

Neglecting radiation absorption in the gas and treating the particle cloud as a continuum, the equations of motion applicable to the analysis are those expressing conservation of mass, momentum, and energy for the particle flow. These may be written

$$(d/dx)(\rho_P u_P) = 0 \tag{5}$$

$$u_P\frac{du_P}{dx} = \frac{C_D\ (\rho/2)(\pi d_P{}^2/4)(u - u_P)|u - u_P|}{\rho_s\ \pi d_P{}^3/6} \tag{6}$$

$$\rho_P u_P c_s \frac{dT_P}{dx} + \frac{dq_r}{dx} = \frac{Nu(\rho_P k/d_P)(\pi d_P{}^2)(T - T_P)}{\rho_s(\pi d_P{}^3/6)} \tag{7}$$

where C_D and Nu will be approximated by[8]

$$C_D = 24/Re/$$
$$1 + (M_R/Re)[3.82 + 1.28\exp(-1.25\ M_R/Re)] \tag{8}$$

$$Nu = 2/[1 + 6.84\ M_R/(RePr)] \tag{9}$$

(Because, even for strong shocks, the relative Mach number will be low subsonic throughout most of the radiation-affected region behind the gas phase discontinuity, and to keep the equations relatively simple, the gas static temperature has been used in place of the recovery temperature in Eq. (7); and the relative Reynolds number in Eqs. (8) and (9) will be evaluated based on the relative freestream condition.)

The net radiative heat flux q_r, appearing in Eq. (7), is taken positive in the direction of increasing x (flow direction); and the differential approximation will be used to represent its transport.[7,9] Thus,

$$d^2q_r/d\xi^2 = 3q_r + 16\sigma T_P{}^3(dT_P/d\xi) \tag{10}$$

where

$$\xi = \int^x \alpha_P(x)dx \tag{11}$$

Equations (1 and 5–11) are sufficient for solution of the problem under consideration. Because the particle temperature gradient and the radius of curvature of the radiative flux profile are discontinuous at the gas phase discontinuity, solutions must be obtained for the upstream and downstream regions and appropriately matched at the location of the discontinuity. Before proceeding further, however, we will recast the equation in nondimensional form. As an intermediate step, Eq. (5) will be integrated to yield $\rho_P u_P = ct = (\rho_P u_P)_{\pm\infty} = \epsilon(\rho u)_{\pm\infty}$. Noting that $u_{P\pm\infty} = u_{\pm\infty}$ so that $\rho_{P\pm\infty} = \epsilon\rho_{\pm\infty}$ and with $T = \bar{T}T_{-\infty}$, $T_P = \bar{T}_P T_{-\infty}$, $u = \bar{u}u_{-\infty}$, $u_P = \bar{u}_P u_{-\infty}$, $q_r = \bar{q}_r(\rho_P u_P c_s T_P)_{-\infty}$, $dx = [(2\rho_s d_P)/(3\rho_P)]d\xi$, the nondimensional forms of Eqs. (6, 7, and 10) become

$$d\bar{u}_P/d\xi = A_+(\bar{u}_{+\infty} - \bar{u}_P) \tag{12}$$

$$d\bar{T}_P/d\xi + d\bar{q}_r/d\xi = B_\pm(\bar{T}_{\pm\infty} - \bar{T}_P) \tag{13}$$

$$d^2\bar{q}_r/d\xi^2 = 3\bar{q}_r + C_\pm \bar{T}_P{}^3\ d\bar{T}_P/d\xi \tag{14}$$

where the subscripts − and + refer to equations that apply to regions upstream and downstream of the gas phase discontinuity, respectively. The constants appearing in these equations are functions of d_P, $M_{-\infty}$, T_0, $p_{-\infty}$, and ϵ, and are given as

$$A_+ = \frac{12/(\epsilon M_{+\infty})}{\{3.82 + (Re_2/M_{+\infty})(\rho_{+\infty}/\rho_{-\infty} - 1) + 1.28\exp[-(1.25\ Re_2/M_{+\infty})/(\rho_{+\infty}/\rho_{-\infty} - 1)]\}}$$

$$B_\pm = \frac{8/(\epsilon c_s/c_P)}{[Re_2 Pr_{-\infty}(\mu_{+\infty}/\mu_{-\infty})(K_{-\infty}/K_{\pm\infty})(\rho_{+\infty}/\rho_{-\infty} - 1) + 6.84\ M_{\pm\infty}]}$$

$$C_\pm = \frac{16/(\epsilon c_s/c_P)}{B_{0-\infty}}$$

The solutions of Eqs. 12–14 are subject to the following boundary and matching conditions,

$$\xi \to -\infty: \quad d^n(\bar{T}_P,\bar{q}_r)/d\xi^n = 0, \quad \bar{T}_P = 1, \quad \bar{u}_P = 1, \quad \bar{q}_r = 0$$

$$\xi = 0: \quad (d\bar{q}_r/d\xi)_1 = (d\bar{q}_r/d\xi)_2, \ \bar{T}_{P_1} = \bar{T}_{P_2}, \quad \bar{u}_{P_1} = \bar{u}_{P_2} = 1, \quad \bar{q}_{r_1} = \bar{q}_{r_2}$$

$$\xi \to +\infty: \quad d^n(\bar{T}_P,\bar{q}_r)/d\xi^n = 0, \quad \bar{T}_P = \bar{T}_{+\infty}, \bar{u}_P = \bar{u}_{+\infty}, \ \bar{q}_r = 0$$

Equation (12) may be solved immediately to yield

$$\bar{u}_{P+} = \bar{u}_{+\infty} - (\bar{u}_{+\infty} - 1) \exp(-A_+\xi) \tag{15a}$$

recalling, of course,

$$\bar{u}_{P-} = 1 \tag{15b}$$

Equations (13) and (14) require simultaneous solution for the dependent variables $\bar{T}_P$ and $\bar{q}_r$ in terms of the independent variable ξ. Although these solutions are independent of $\bar{u}_P$ in the ξ plane, the solution for $\bar{u}_P$ is necessary to transform them to the x plane. Before proceeding with the discussion of the numerical solution of these equations, it will prove instructive to consider an approximate solution.

Approximate Solution

If we assume $B_\pm$ to be large compared with $C_\pm$, then the particle temperature, except in the thin region where T_P adjusts to the gas temperature behind the gas phase discontinuity, is nearly uniform in the radiation-dominated regions. Thus, from Eq. (14),

$$d^2\bar{q}_r/d\xi^2 \approx 3\bar{q}_r$$

or

$$\bar{q}_{r-} \approx D_1 e^{\xi(3)^{1/2}}, \ \bar{T}_{P-} \approx 1; \quad -\infty \leq \xi \leq 0$$

$$\bar{q}_{r+} \approx D_2 e^{-\xi(3)^{1/2}}, \ \bar{T}_{P+} \approx \bar{T}_{+\infty}; \quad \delta \leq \xi \leq +\infty; \quad |\delta| \ll 1$$

The constants D_1 and D_2 may be determined by assuming that insignificant absorption occurs in the region $0 \leq \xi \leq \delta$, so that $D_1 \approx D_2 = D$ and

$$(dq_r/d\xi)|_\delta - (dq_r/d\xi)|_1 \approx (C_\pm/4)(\bar{T}_{+\infty}{}^4 - 1)$$

from which

$$D \approx -[C_\pm/8(3)^{1/2}](\bar{T}_{+\infty}{}^4 - 1)$$

Thus,

$$\bar{q}_{r\mp} \approx -[C_\mp/8(3)^{1/2}](\bar{T}_{+\infty}{}^4 - 1)e^{\pm\xi(3)^{1/2}}; \quad C_\mp \ll B_\mp \tag{16}$$

Perturbations in $\bar{T}_P$ within the range of validity of Eq. (16) may be obtained from solution of Eq. (13) with the aid of Eq. (16). Thus we find

$$\bar{T}_{P-} \approx 1 + \frac{C_-}{8[B_- + (3)^{1/2}]}(\bar{T}_{+\infty}{}^4 - 1)e^{\xi(3)^{1/2}}; \ C_\mp \ll B_\mp \tag{17a}$$

$$\bar{T}_{P+} \approx \bar{T}_{P+\infty} + \left\{1 - \bar{T}_{+\infty} + \frac{C_+}{8}(\bar{T}_{+\infty}{}^4 - 1) \times \left[\frac{1}{B_- + (3)^{1/2}} + \frac{1}{B_+ - (3)^{1/2}}\right]\right\}e^{-B_+\xi} - \frac{C_+}{8[B_+ - (3)^{1/2}]}(\bar{T}_{+\infty}{}^4 - 1)e^{-\xi(3)^{1/2}}; \quad C_\mp \ll B_\mp \tag{17b}$$

Inspection of Eqs. (17a) and (17b) reveals that, as initially noted, $\bar{T}_P$ is nearly uniform in the regions upstream and downstream of the gas phase discontinuity when $C_\mp \ll B_\mp$. We may also note that the radiative effect on $\bar{T}_P$ will increase for increasing C and/or decreasing B; i.e., with increasing T_0 and d_P and with decreasing $\epsilon c_s/c_P$ and $p_{-\infty}$. The results of numerical solutions, to be described next, will quantify the relative influence of such variations on the shock structure.

Numerical Solution

To facilitate solution, Eqs. (13) and (14) can be rewritten as a system of first-order ordinary differential equations as

$$\frac{d\bar{q}_r}{d\xi} = \bar{\eta} \tag{18}$$

$$\frac{d\bar{T}_P}{d\xi} = B_\pm(\bar{T}_{\pm\infty} - \bar{T}_P) - \bar{\eta} \tag{19}$$

$$\frac{d\bar{\eta}}{d\xi} = 3\bar{q}_r + (BC)_\pm \bar{T}_P{}^3(\bar{T}_{\pm\infty} - \bar{T}_P) - C_\pm \bar{T}_P{}^3\bar{\eta} \tag{20}$$

Because Eqs. 18–20 are singular at the boundaries ($\xi \to \pm\infty$), numerical solution by marching techniques requires the assumption of small perturbations in $\bar{T}_P$, $\bar{q}_r$, and $\bar{\eta}$ at those locations. Since these variables are related, such perturbations cannot be independently chosen. In the present case they were determined by dividing Eqs. (18) and (20) by Eq. (19) and evaluating the resulting equations at $\xi \to \pm\infty$ with the aid of L'Hopital's rule to remove the indeterminacies. Thus, we find

$$\left(\frac{d\bar{q}_r}{d\bar{T}_P}\right)_{\pm\infty} = \frac{-(d\bar{\eta}/d\bar{T}_P)_{\pm\infty}}{B_\pm + (d\bar{\eta}/d\bar{T}_P)_{\pm\infty}} \tag{21}$$

$$\left(\frac{d\bar{\eta}}{d\bar{T}_P}\right)_{\pm\infty} = \frac{-3(d\bar{q}_r/d\bar{T}_P)_{\pm\infty}}{B_\pm + (d\bar{\eta}/d\bar{T}_P)_{\pm\infty}} + C_\pm T_{\pm\infty}{}^3 \tag{22}$$

Solving Eqs. (21) and (22) we have

$$\left(\frac{d\bar{\eta}}{d\bar{T}_P}\right)^3_{\pm\infty} + a_\pm\left(\frac{d\bar{\eta}}{d\bar{T}_P}\right)^2_{\pm\infty} + b_\pm\left(\frac{d\bar{\eta}}{d\bar{T}_P}\right)_{\pm\infty} + c_\pm = 0 \tag{23}$$

where

$$a_\pm = 2B_\pm - C_\pm\bar{T}_{\pm\infty}{}^3; \quad b_\pm = B_\pm{}^2 - 2B_\pm C_\pm \bar{T}_{\pm\infty}{}^3 - 3$$
$$C_\pm = -B_\pm{}^2 C_\pm \bar{T}_{\pm\infty}{}^3$$

Equations (23) and (21) may be used to find $(d\bar{\eta}/dT_P)_{\pm\infty}$ and $(d\bar{q}_r/d\bar{T}_P)_{\pm\infty}$. Since three solutions are possible, those that are physically meaningful are those for which $(d\bar{q}_r/d\bar{T}_P)_{-\infty}$, $(d\bar{\eta}/d\bar{T}_P)_{-\infty} < 0$ and $(d\bar{q}_r/d\bar{T}_P)_{+\infty} > 0$, $(d\bar{\eta}/d\bar{T}_P)_{+\infty} < 0$. Thus, perturbations in $\bar{T}_P$ at $\pm\infty$ may be used to estimate corresponding perturbations in $\bar{q}_r$ and $\bar{\eta}$.

Two methods of solution of Eqs. 21–23 were attempted. In the first, solutions were obtained by marching forward and backward from the upstream and downstream boundaries with $\xi = 0$ corresponding to the position where $\bar{\eta}$, $\bar{T}_P$, and $\bar{q}_r$ from both solutions were simultaneously matched. The backward marching solution proved to be unstable, however, and the first method was discarded. The second method, though more tedious, but which provided stable solutions, consisted of marching forward from the upstream boundary, guessing the location of $\xi = 0$, and continuing to march to the downstream boundary, the proper solution being obtained when the downstream boundary conditions were satisfied.

Results and Discussion

Numerical calculations were carried out to demonstrate the effects of independent variations of pressure, particle loading, particle diameter, stagnation temperature, and shock Mach number. The gas phase was assumed to be air ($\gamma = 1.4$) and the flow conditions at which solutions were obtained are summarized in Table 1.

The calculated results are presented in Fig. 2 in comparison with corresponding approximate predictions obtained through use of Eqs. (16) and (17). It was not considered necessary to

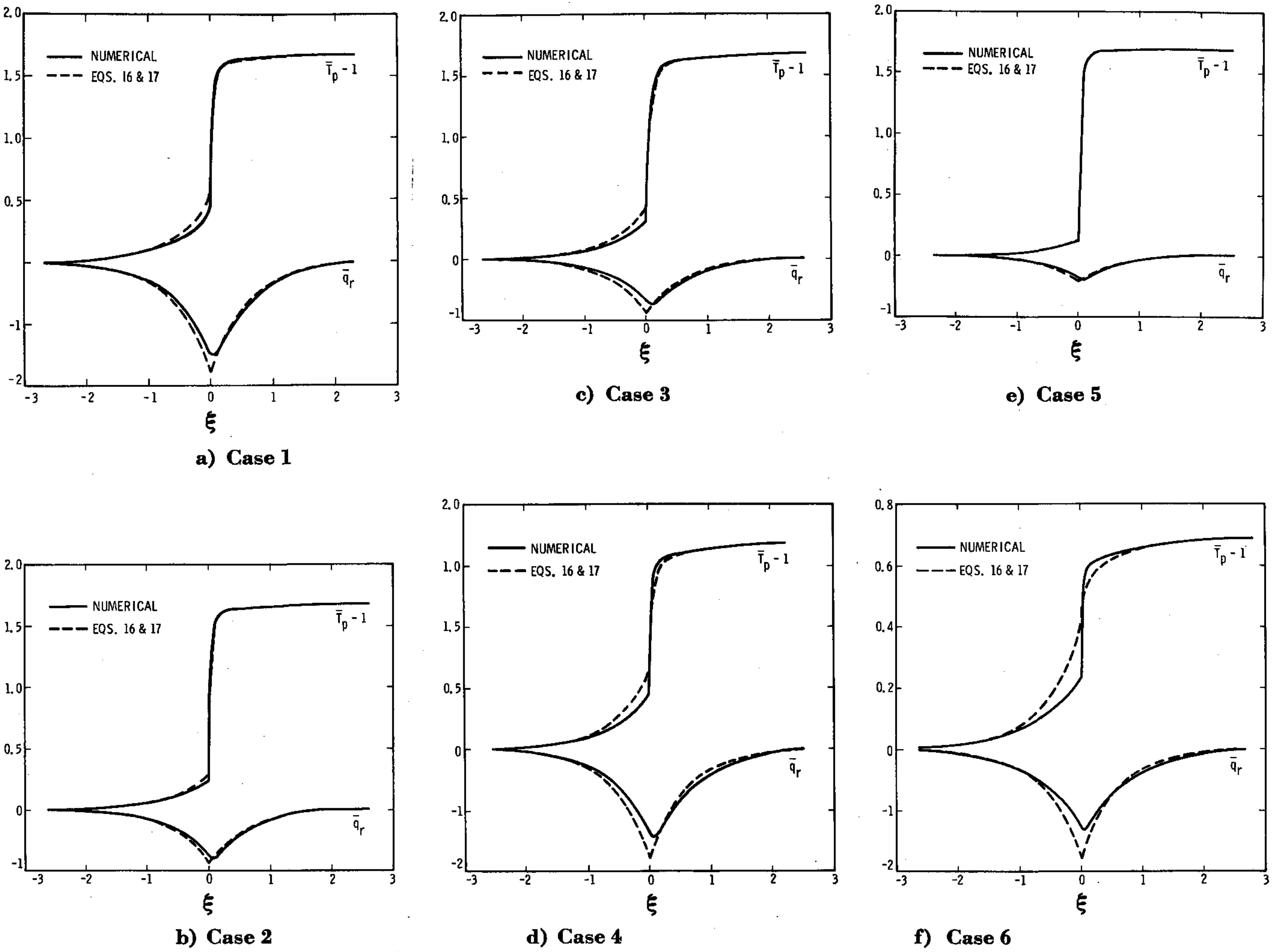

Fig. 2 Particle temperature and radiative flux distributions for radiation-dispersed shock waves in gas-particle flows.

transform these solutions to the physical plane in the present study. An indication of the extent of radiation-dominated region can be obtained, however, from values of $(c_P/\rho_s c_s)(dx/d\xi)$ given in Table 1 for the region upstream of the gas phase discontinuity and at downstream infinity. Values for the remaining region will range between these two quantities. It may be noted in all cases the influence of radiation on particle temperature extends large distances (order of 10^6 particle diameters) upstream of the gas phase discontinuity.

In general, for the range of flow conditions numerically investigated, the results are in reasonable agreement with those of the approximate theory, the worst deviation occurring in case 6. (Recalling that the approximate theory requires $C \ll B$, and noting that, in case 6, the value of C/B is an order of magnitude higher than those of the other cases, the difference between the numerical result and that of the approximate theory is understandable.) The results of case 1 will be used as reference against which the results of independent variations in the flow parameters, represented in the remaining cases, can be compared. Because of the relatively good agreement with the approximate theory, Eqs. (16) and (17) are useful in explaining the results of such variations. In case 1, approximately 26% of the particle temperature rise across the shock occurred upstream of the gas phase discontinuity; and the maximum absolute nondimensional radiative flux, representing the ratio of radiative flux to upstream particle flow enthalpy flux, was about 1.47.

Effect of $P_{-\infty}$ (case 2). Doubling the upstream pressure nearly halved the particle temperature rise approaching the gas phase discontinuity, primarily due to a doubling of the Boltzmann number resulting in a halving of $C_{\pm}$. The extent of the radiation-influenced region (physical plane) was also halved.

Effect of $\epsilon c_s/c_P$ (case 3). Doubling $\epsilon c_s/c_P$ halved both $C_{\pm}$ and $B_{\pm}$, lowering the upstream particle temperature rise by approximately 25%. The extent of the radiation-influenced region was halved.

Effect of d_P (case 4). A tenfold increase in particle size had no effect on $C_{\pm}$, produced minor decreases in B_-, and a one-third reduction in B_+. Although only a slight increase in

Table 1 Flow conditions for shock structure calculations

Case	$M_{-\infty}$	$T_0(°K)$	$d_P(\mu)$	$\epsilon c_s/c_P$	$P_{-\infty}$(atm)	B_-	B_+	$C_{\pm}$	$[(c_P/\rho_s c_s)dx/d\xi]$ $(-)$	(cm/gm/cc) $(+\infty)$
1	3	3333	1	0.1	0.05	3.82	23.30	0.492	45.0	11.7
2	3	3333	1	0.1	0.10	3.75	22.10	0.246	22.5	5.8
3	3	3333	1	0.2	0.05	1.91	11.65	0.246	22.5	5.8
4	3	3333	10	0.1	0.05	3.25	15.48	0.492	450.0	116.6
5	3	2222	1	0.1	0.05	3.78	22.50	0.119	30.0	7.8
6	2	3333	1	0.1	0.05	5.78	19.80	3.480	70.0	26.2

the upstream particle temperature rise resulted, the extent of the radiation-influenced region was increased by an order of magnitude.

Effect of T_0 (case 5). A one-third reduction in T_0 produced an approximately fivefold increase in the Boltzmann number thereby decreasing $C_\pm$ by nearly 80%. Only minor reduction in $B_\pm$ occurred. Consequently, the particle temperature rise approaching the gas phase discontinuity was reduced by approximately 75%, and the extent of the radiation-influenced region was reduced by one-third.

Effect of $M_{-\infty}$ (case 6). Reducing the shock Mach number from 3 to 2 produced a nearly tenfold increase in $C_\pm$ and less significant changes in $B_\pm$. In this case the fraction of the particle temperature rise across the shock occurring upstream of the gas phase discontinuity increased to 35%, and the extent of the radiation-influenced region increased by approximately 50%.

Finally, to determine if, under the conditions, calculations were made, the gas flow was significantly affected by the particle flow, and to assess the error such an influence would have on the results presented, Eq. (17a) was combined with the gas phase energy equation to estimate the gas temperature rise in the region upstream of the gas phase discontinuity. This result was then used in Eq. (17a) to calculate an improved estimate of the particle temperature rise in that region. Thus,

$$(T_- - 1)/(T_P - 1) \approx 1 + [\epsilon c_s/c_P]/[\epsilon c_s/c_P + (3)^{1/2}/B_-]$$

$$\frac{(T_{P-} - 1)\ \text{actual}}{(T_{P-} - 1)\ \text{presented}} \approx \frac{\epsilon c_s/c_P}{[B_- + (3)^{1/2}](\epsilon c_s/c_P + (3)^{1/2}/B_-)}$$

The first equation indicates the upstream gas temperature rise was less than 18% of the corresponding particle temperature rise in cases 1–5 and increased to 25% in case 6. (The defining equation for B_- shows this fraction will fall below 10% for $M_{-\infty} \geq 6$). The second equation demonstrates the upstream particle temperature rise presented is approximately 3% low in cases 1–5 and 4% low in case 6. Thus, in spite of the fact that the upstream gas temperature rise was not entirely insignificant, the affect of such a rise on the results presented is negligible.

Conclusions

Within the limits of the assumptions imposed upon the present analysis, the following conclusions can be made. 1) Thermal radiation can have a significant effect on the structure of normal shock waves in gas-particle flows at conditions where gas radiation is negligible. 2) The extent of the region influenced by thermal radiation can extend large distances upstream of the gas phase discontinuity. 3) The effects of thermal radiation increase with increasing stagnation temperature and particle size and decreasing pressure and particle loading.

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